Extension Tubes and Effective f-stops

The purpose of an extension tube is to increase the lens-to-film distance to allow for a closer approach to the subject. This, in turn, magnifies the image. Accordingly, there are two competing effects that contribute to the change in film exposure --- a closer approach to the subject increases the light intensity striking the film (inverse square law), while image magnification spreads light over a wider area, thereby reducing the light intensity at the film. Let's quantify these effects.

Figure 1 shows an object a distance s_1 in front of a thin lens of focal length f and its image at the film plane a distance i_1 behind the lens. The object and image heights are h and h_1 respectively. Let's suppose that the lens is on a camera and is racked out to its closest focus position.

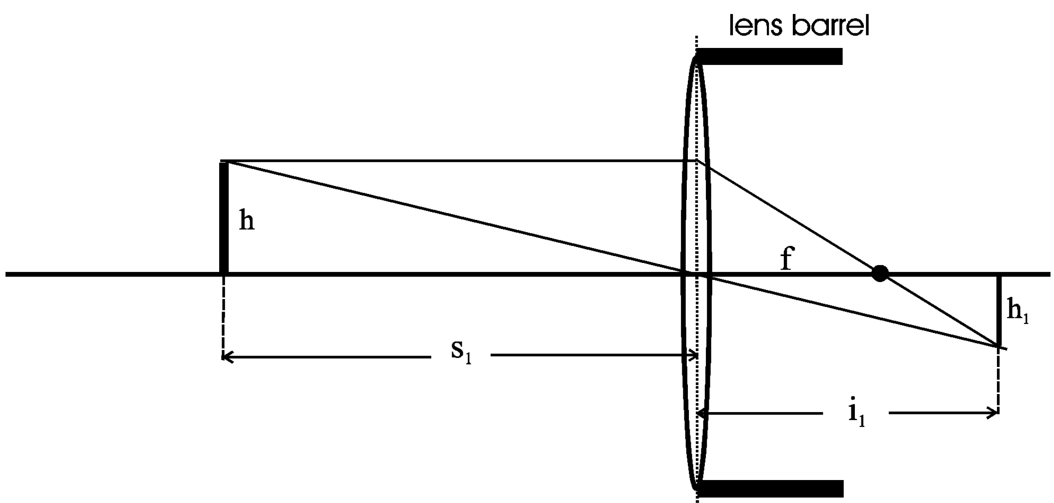

Figure 1.

The object and image distances are related by the thin-lens formula:

$$\frac{1}{s_1} + \frac{1}{i_1} = \frac{1}{f} \tag{1}$$

which can be rearranged to give

$$\frac{i_1}{s_1} = \frac{i_1 - f}{f} \quad \text{or} \quad \frac{i_1}{s_1} = \frac{f}{s_1 - f} \tag{2}$$

The magnification ratio is the image height divided by the object height. By similar triangles, and using the first expression in Equation 2, it is

$$m_1 = \frac{h_1}{h} = \frac{i_1}{s_1} = \frac{i_1 - f}{f} \tag{3}$$

Combining this with the second relation in Equation 2, we get (for later use)

$$s_1 = \frac{m_1 + 1}{m_1} f \tag{4}$$

Suppose that this configuration doesn't give enough magnification. For example, the image of a nudibranch occupies a small central portion of the frame instead of filling it. A simple way to overcome this problem is to insert an extension tube of length L between the camera body and the lens, as shown in Figure 2.

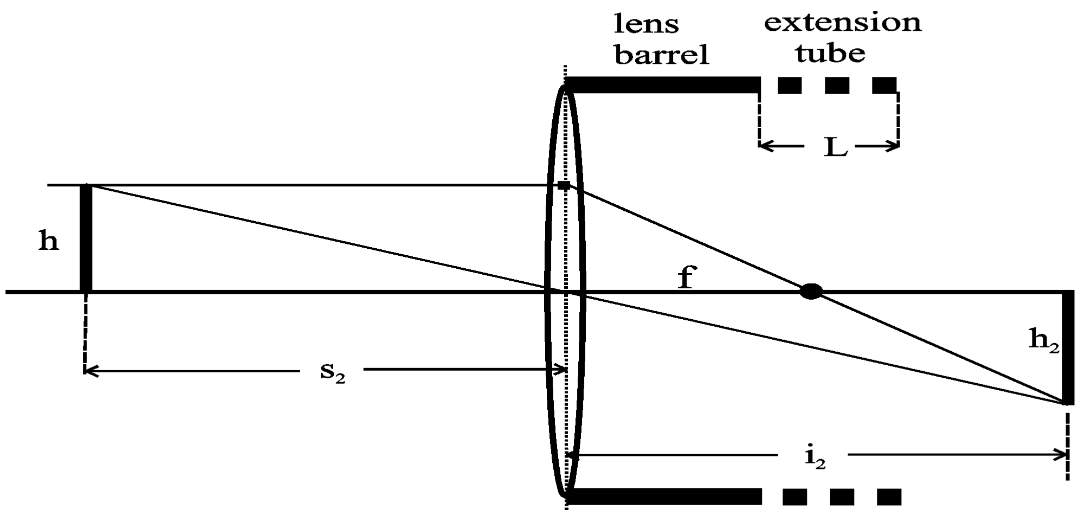

Figure 2.

This increases the lens-to-film distance to the value

$$i_2 = i_1 + L \tag{5}$$

which requires a **decrease** in the lens-to-subject distance in order to maintain focus at the film plane. Because of this, the image gets magnified, and the new magnification ratio is

$$m_2 = \frac{h_2}{h} = \frac{i_2}{s_2} \tag{6}$$

By analogy with the first expression in Equation 2, this becomes

$$\begin{aligned} m_2 &= \frac{i_2 - f}{f} \\ &= \frac{i_1 - f}{f} + \frac{L}{f} \end{aligned} \tag{7}$$

or, using Equation 3,

$$m_2 = m_1 + \frac{L}{f} \tag{8}$$

The analogue of Equation 4 for this new configuration is

$$s_2 = \frac{m_2 + 1}{m_2} f \tag{9}$$

The insertion of the extension tube has caused the image height to increase by the factor

$$\frac{h_2}{h_1} = \frac{m_2 h}{m_1 h} = \frac{m_2}{m_1} \tag{10}$$

But **al**l dimensions of the image are scaled by this factor. This means that areas are scaled by the **square** of this factor. In particular, that tiny nudibranch in the middle of the film frame now spans the entire frame, occupying an area that is larger by the factor $(m_2/m_1)^2$. Correspondingly, if there is no change in the light intensity reflected by the nudibranch and **entering** the aperture, the magnification of the image reduces the intensity of the light at the film plane by the factor $(m_1/m_2)^2$.

But there’s another consideration. The lens-to-subject distance has been reduced from s_1 to s_2, so by the inverse square law the intensity of the light passing through the aperture must have increased by the factor $(s_1/s_2)^2$. The overall change in the light intensity at the film plane is therefore the product of the two factors. This **exposure factor** is

$$EF = \left(\frac{m_1}{m_2}\right)^2 \left(\frac{s_1}{s_2}\right)^2 \tag{11}$$

But s_1 and s_2 are given in Equations 4 and 9 in terms of m_1 and m_2 respectively. Substituting those expressions into Equation 9, the exposure factor becomes

$$EF = \left(\frac{m_1 + 1}{m_2 + 1} \right)^2 \qquad (12)$$

Note that m_2 is larger than m_1, so EF<1 and there is a net **reduction** in light intensity at the film plane. It's **as if** the insertion of the extension tube has decreased the size of the aperture. It hasn't really, but if we pretend it has then we can fool the camera or hand-held light meter into giving more light and producing a good exposure. We need to know what to tell the light meter.

Let's assume that the required f-stop for a perfect exposure without the extension tube was

$$F = \frac{f}{d} \qquad (13)$$

where d is the diameter of the aperture. With the tube in place, the exposure reduction is equivalent to a reduction in the area of the aperture by the factor EF, or a reduction in the diameter of the aperture by the square root of EF. This produces the **effective f-stop**

$$\begin{aligned} F_{eff} &= \frac{f}{d\sqrt{EF}} \\ &= \left(\frac{m_2 + 1}{m_1 + 1} \right) F \end{aligned} \qquad (14)$$

Usually, $m_1 \ll 1$, so to a good approximation this is

$$F_{eff} = (m_2 + 1) F \qquad (16)$$

As long as L is not very small compared with f, Equation 8 for m_2 gives

$$\begin{aligned} F_{eff} &= \left(1 + \frac{L}{f} \right) \frac{f}{d} \\ &= \frac{f + L}{d} \end{aligned} \qquad (17)$$

This result shows that for lenses with small reproduction ratios, the effective f-stop is obtained by using an "effective" focal length, which is the sum of the lens' actual focal length and the length of the extension tube.

A common example is the case of 1:1 reproduction with an extension tube. In this case m_2=1 and (by Equation 16) F_{eff}=2F. If the f-stop is set to 22 you'd better pretend that it's 44.